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FATIGUE RELIABILITY ANALYSIS OF COMPOSITE LAMINATES UNDER SPECTRUM STRESS

T. Y. KAM, S. Y. TSAI and K. H. CHU
Department of Mechanical Engineering, National Chiao Tung University, Hsin Chu 300, Taiwan, Republic of China

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Abstract—Fatigue reliability of graphite/epoxy composite laminates under uniaxial spectrum stress is studied using the modified β-method. A number of cumulative damage models are adopted to construct the limit state equation for the laminates in the reliabilty analysis. Statistics of fatigue life distributions of the laminates under cyclic stresses of constant amplitudes obtained from tests are used for the reliability assessment of the laminates under spectrum stress. The accuracy of the modified β-method in fatigue reliability prediction is then verified by experimental multi-stress level fatigue life data. The applicabilities of the modified β-method and the cumulative damage models in fatigue reliability prediction are discussed. It has been found that several of the adopted cumulative damage models may be applicable for fatigue reliability analysis of composite laminates under spectrum stress. © 1997 Elsevier Science Ltd. All rights reserved.

1. INTRODUCTION

Recently laminated composite materials have been used extensively in the construction of high performance structures such as mechanical, automotive, marine and aerospace structures which also require high reliability. In general these structures, which are subjected to dynamic loads, are susceptible to fatigue failure. In order to avoid catastrophic failure, fatigue behavior of laminated composite materials has drawn close attention and become an important topic of research in recent years. In practice, for structures subjected to constant amplitude stress fatigue, emperical relations such as Basquin's relation, S–N curve and Coffin–Manson's relation (Collins, 1981) are generally used for predicting fatigue life of laminated composite materials. On the other hand, for structures subjected to varying or random loadings, cumulative damage rules are commonly used for fatigue life prediction. Numerous studies on cumulative damage models and fatigue life prediction have been reported in the literature (e.g. Miner, 1945; Owen and Howe, 1972; Henry, 1954). A review of the cumulative damage models applicable for fatigue life prediction of composite materials has also been performed by Hwang and Han (1986). Since dispersion of fatigue life of composite materials is inevitable, accurate prediction of fatigue life then becomes intractable. Therefore, the prediction of fatigue life should be treated in a probabilistic, rather than a deterministic, way if the design of a composite structure of high reliability is desired.

Recently, many efforts have been devoted to the study of fatigue life scattering and a number of probabilistic models have been proposed for modeling fatigue life distributions (e.g. Johnsen and Doner, 1981; Han and Hamddi, 1983; Yang *et al.*, 1992). It is not difficult to realize that the previous studies have been mainly concentrated on laminates subjected to cyclic stress of constant amplitude. For instance, Radhakrishnan (1984) used Weibull distribution to study the fatigue life of composite material specimens with zero degree fiber angle after proof test. As for fatigue reliability of materials subjected to spectrum stress, only limited work has been done in this area. For instance, Collins (1981) used cumulative damage rules and probabilistic S–N curves to study fatigue reliability of materials under spectrum stress. Yang and Du (1983) used a statistical model, which was constructed on the basis of a fatigue and residual strength degradation model for constant amplitude cyclic loading, to study fatigue life distribution of composite materials subject to service loading spectra.

In this paper, the modified β-method (Ang and Tang, 1984) which is formulated on the basis of the structural relaibility theory is used to study the multi-stress level fatigue reliability of composite laminates. Different cumulative damage models are adopted in the construction of the limit state equation of the composite laminates. Experimental investigation of fatigue life distribution of $[45°/-45°_2/45°]_s$ composite specimens is performed. Experimental data are used to study the feasibility of the modified β-method and the applicability of the cumulative damage models.

2. CUMULATIVE DAMAGE MODELS

Fatigue damage of composite materials depends on many factors such as applied stress level, number of fatigue cycles, frequency, temperature, etc. Without loss of generality, only the effects of applied stress level and number of fatigue cycles on fatigue damage are considered in this study. The damage can then be written in a functional form as

$$D = F(n, r) \tag{1}$$

where n is number of stress cycles; r is stress ratio.

$$r = \frac{S_{\max}}{S_u} \tag{2}$$

where $S_{\max}$ is the maximum value of the applied stress, S_u is the ultimate stress.

For a constant amplitude stress, the damage is bound by the following conditions,

$$\begin{aligned} D = 0 \quad &\text{when} \quad n = 0 \\ D = 1 \quad &\text{when} \quad n = N \end{aligned} \tag{3}$$

where N is fatigue life under cyclic stress of constant amplitude.

The functional form in eqn (1) can be classified into two categories, namely, linear and nonlinear damage models. The linear model of Palmgren–Miner's rule and several nonlinear models will be adopted in the following fatigue reliability analysis of composite laminates. Brief descriptions of the models are given as follows:

(a) *Palmgren–Miner's rule*. Palmgren–Miner's rule is a linear damage rule which defines damage as the ratio of the number of applied stress cycles n and the number of cycles to failure N, i.e.

$$D = \frac{n}{N}. \tag{4}$$

(b) *Modified Palmgren–Miner's rule*. The damage is defined as

$$D = \left(\frac{n}{N}\right)^c \tag{5}$$

where c is a constant.

(c) *Hwang–Han's model I (H-H-I)*. Fatigue damage is defined as the ratio of the resultant strain at the nth cycle, $\varepsilon(n)$, and the failure strain, ε_f.

$$D = \frac{\varepsilon(n)}{\varepsilon_f}. \tag{6}$$

Using the concept of fatigue modulus degradation proposed by Hwang and Han (1986), it can be shown that the above equation can be written as

$$D = \frac{(\mathrm{B} - N^c)}{(\mathrm{B} - n^c)} \tag{7}$$

where B and c are constants.

(d) *Hwang–Han's model II (H-H-II)*. Herein fatigue damage is defined as

$$D = \frac{\varepsilon(n) - \varepsilon_0}{\varepsilon_f - \varepsilon_0} \tag{8}$$

where ε_0 is the strain under the maximum stress S_{max} at the first fatigue cycle. Based on the concept of fatigue modulus, the above fatigue damage can be written as

$$D = \left(\frac{n}{N}\right)^c \left[\frac{\mathrm{B} - N^c}{\mathrm{B} - n^c}\right]. \tag{9}$$

When a specimen is subjected to a spectrum stress which is comprised of a number of stress levels of different amplitudes, the total damage is obtained as

$$D = \sum_{i=1}^{m} \Delta D_i \tag{10}$$

where m is number of stress levels, ΔD_i is damage accumulated during stress level r_i. Failure occurs when the total damage reaches or exceeds 1, i.e.

$$D \geqslant 1. \tag{11}$$

The determination of ΔD_i is based on the concept of equivalent damage. The case of two-stress level fatigue is used to illustrate the procedure for determining ΔD_i. Let n_{12} be the number of cycles at the stress level r_2, which has an equivalent damage under the stress level r_1 for n_1 cycles. The determination of n_{12} is achieved by equating ΔD_1 and ΔD_{12}.

$$\Delta D_1 = \Delta D_{12} \tag{12}$$

or

$$F\left(\frac{n_1}{N_1}\right) = F\left(\frac{n_{12}}{N_2}\right). \tag{13}$$

Here n_{12} is obtained by solving the above equation. The fatigue damage induced directly by r_2, ΔD_2, is then obtained as

$$\Delta D_2 = F\left(\frac{n_2 + n_{12}}{N_2}\right) - F\left(\frac{n_{12}}{N_2}\right). \tag{14}$$

The procedure of damage determination for two-stress level can be extended to multi-stress level cases. For illustration, the fatigue damage of two- or three-stress levels derived for different cumulative damage models are given as follows.

2.1. *Two-stress level*

(a) Palmgren–Miner's rule

$$D = \frac{n_1}{N_1} + \frac{N_2}{N_2}. \tag{15}$$

(b) Modified Palgren–Miner's rule

$$D = \left(\frac{n_1}{N_1} + \frac{n_2}{N_2}\right)^c. \tag{16}$$

(c) H-H-I

$$D = \frac{\mathrm{B} - N_1^c}{\mathrm{B} - n_1^c} + \frac{\mathrm{B} - N_2^c}{\mathrm{B} - (n_{12} + n_2)^c} - \frac{\mathrm{B} - N_2^c}{\mathrm{B} - n_{12}^c} \tag{17a}$$

with

$$n_{12} = \left\{\mathrm{B}\left[1 - \left(\frac{r_2}{r_1}\right)\left(1 - \frac{n_1^c}{\mathrm{B}}\right)\right]\right\}^{1/c}. \tag{17b}$$

(d) H-H-II

$$D = \left(\frac{n_1}{N_1}\right)^c \left[\frac{(\mathrm{B} - N_1^c)}{(\mathrm{B} - n_1^c)}\right] + \left(\frac{n_{12} + n_2}{N_2}\right)^c \left[\frac{(\mathrm{B} - N_2^c)}{(\mathrm{B} - (n_{12} + n_2)^c)}\right] - \left(\frac{n_{12}}{N_2}\right)^c \left[\frac{(\mathrm{B} - N_2^c)}{(\mathrm{B} - n_{12}^c)}\right] \tag{18a}$$

with

$$n_{12} = \left\{\frac{1}{\mathrm{B}}\left[1 + \left(\frac{r_2}{1 - r_2}\right)\left(\frac{1 - r_1}{r_1}\right)(\mathrm{B}n_1^{-c} - 1)\right]\right\}^{-1/c}. \tag{18b}$$

2.2. *Three-stress level*

(a) Palmgren–Miner's rule

$$D = \frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3}. \tag{19}$$

(b) Modified Palmgren–Miner's rule

$$D = \left(\frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3}\right)^c. \tag{20}$$

(c) H-H-I

$$D = \frac{\mathrm{B} - N_1^c}{\mathrm{B} - n_1^c} + \left[\frac{\mathrm{B} - N_2^c}{\mathrm{B} - (n_{12} + n_2)^c} - \frac{\mathrm{B} - N_2^c}{\mathrm{B} - n_{12}^c}\right] + \left[\frac{\mathrm{B} - N_3^c}{\mathrm{B} - (n_{23} + n_3)^c} - \frac{\mathrm{B} - N_3^c}{\mathrm{B} - n_{23}^c}\right] \tag{21a}$$

with

$$n_{23} = \left\{ \mathrm{B}\left[1 - \left(\frac{r_3}{r_2}\right)\left(1 - \frac{(n_{12}+n_2)^c}{\mathrm{B}}\right)\right]\right\}^{1/c} \tag{21b}$$

and n_{12} is the same as eqn (17b).

(d) H-H-II

$$D = \left(\frac{n_1}{N_1}\right)^c \left[\frac{(\mathrm{B}-N_1^c)}{(\mathrm{B}-n_1^c)}\right] + \left(\frac{n_{12}+n_2}{N_2}\right)^c \left[\frac{(\mathrm{B}-N_2^c)}{(\mathrm{B}-(n_{12}+n_2)^c)}\right] - \left(\frac{n_{12}}{N_2}\right)^c \left[\frac{(\mathrm{B}-N_2^c)}{(\mathrm{B}-n_{12}^c)}\right]$$
$$+ \left(\frac{n_{23}+n_3}{N_3}\right)^c \left[\frac{\mathrm{B}-N_3^c}{\mathrm{B}-(n_{23}+n_3)^c}\right] - \left(\frac{n_{23}}{N_3}\right)^c \left[\frac{\mathrm{B}-N_3^c}{\mathrm{B}-n_{23}^c}\right] \tag{22a}$$

with

$$n_{23} = \left\{ \frac{1}{\mathrm{B}}\left[1 + \left(\frac{r_3}{1-r_3}\right)\left(\frac{1-r_2}{r_2}\right)(B(n_{12}+n_2)^{-c}-1)\right]\right\}^{-1/c} \tag{22b}$$

and n_{12} is the same as eqn (18b).

3. FATIGUE RELIABILITY ANALYSIS

The fatigue reliability of composite laminates is studied on the basis of the structural reliability theory which states that a structural system fails to function properly when it reaches a limit state and the probability that the system does not reach the limit state is defined as the reliability of the system. The limit state in terms of n random variables, x_1, $x_2, \ldots, x_n$, may be denoted as

$$g(x_1, x_2, \ldots, x_n) = 0. \tag{23}$$

The failure probability of this limit state can be written as

$$P_f = \int, \ldots, \iint_{g<0} f_{x_1, x_2, \ldots, x_n}(x_1, x_2, \ldots, x_n)\mathrm{d}x_1, \ldots, \mathrm{d}x_n \tag{24}$$

in which P_f is failure probability; $f_{x_1, x_2, \ldots, x_n}(\cdot)$ is the joint probability density function; the integration is performed over the region where $g < 0$. The reliability of the system, P_s, is obtained as

$$P_s = 1 - P_f. \tag{25}$$

In many instances it is difficult to solve the integration of eqn (24) analytically due to the nonlinearity of the limit state equation. For this reason, the first-order second-moment (β) method (Hasofer and Lind, 1976) is adopted to estimate the failure probability of eqn (24). For the purpose of completeness, a brief description of the method is given as follows. In the first-order second-moment method, the random variables are characterized by their first- and second-moments and the limit state equation is linearized at some point for purposes of performing the reliability analysis. Linearization of the limit state equation leads to

$$g(x_1,\ldots,x_n) = g(x_1^*,\ldots,x_n^*)+\Sigma(x_i-\bar{x}_i)\cdot\left(\frac{\partial g}{\partial x_i}\right)_{x^*} \tag{26}$$

where $(x_1^*, x_2^*, \ldots, x_n^*)$ is the linearizing point. The reliability analysis is then performed with respect to the above linearized limit state equation. The selection of an appropriate linearization point is an important consideration. The selection procedure starts by transforming the variables x_i to reduced variables with zero mean and unit variance through

$$u_i = \frac{x_i-\bar{x}_i}{\sigma_i} \tag{27}$$

where $\bar{x}_i$, σ_i are mean and standard deviation of x_i, respectively. In the space of reduced coordinates u_i, the limit state is

$$g_1(u_1, u_2, \ldots, u_n) = 0 \tag{28}$$

with failure occurring when $g_1 < 0$. Next define a reliability index β as the shortest distance between the surface $g_1 = 0$ and the origin. The point $(u_1^*, u_2^*, \ldots, u_n^*)$ on $g_1(\cdot) = 0$ which corresponds to this shortest distance is referred to as the design point and must be determined by solving the system of equations

$$\alpha_i = \frac{\dfrac{\partial g_1}{\partial u_i}}{\left[\Sigma\left(\dfrac{\partial g_1}{\partial u_i}\right)^2\right]^{1/2}} \tag{29}$$

$$u_i^* = -\alpha_i\beta \tag{30}$$

and

$$g_1(u_1^*, u_2^*, \ldots, u_n^*) = 0 \tag{31}$$

searching for the direction cosines α_i which minimize β. The derivatives $\delta g_1/\delta u_i$ are evaluated at the point $(u_1^*, u_2^*, \ldots, u_n^*)$. In the original variable space, the design point variables are given by

$$x_i^* = \bar{x}_i-\alpha_i\beta\sigma_i \tag{32}$$

and

$$g(x_1^*, x_2^*, \ldots, n_n^*) = 0. \tag{33}$$

The design point and the corresponding distance between the origin and the design point β are then obtained via an iteration procedure. If $x_i (i=1,\ldots,n)$ are independent normal variates, the failure probability of the system can be determined once β is available.

$$P_f = \Phi(-\beta) \tag{34}$$

where $\Phi(\cdot)$ is the cumulative distribution of the standard normal variate. If $x_i (i=1,\ldots,n)$ are non-normal variables, the modified β-method (Ang and Tang, 1984) is used and a transformation of the non-normal variables into equivalent normal variables must be performed prior to the solution of eqns (29)–(31). This transformation may be accomplished by approximating the true distribution of variable x_i by a normal distribution to the value x_i^* corresponding to a point on the failure surface. The mean and standard deviation of the equivalent normal variable for the point x_i^* where the cumulative probability and probability density of the actual and approximating normal variables are equal are obtained as

$$\sigma_i^N = \frac{\phi(\Phi^{-1}[F_i(x_i^*)])}{f_i(x_i^*)} \tag{35a}$$

and

$$\bar{x}_i^N = x_i^* - \Phi^{-1}[F_i(x_i^*)]\sigma_i^N \tag{35b}$$

in which F_i and f_i are non-normal distribution and density functions of x_i; $\phi(\cdot)$ is the density function for the standard normal variate. Having determined $\bar{x}_1^N$ and σ_i^N of the equivalent normal distributions, the solution proceeds exactly as described in eqns (27)–(31). In as much as the design point variable x_i^* changes with each iteration, the parameters $\bar{x}_i^N$ and σ_i^N must be recomputed during each iteration cycle also. The following summarizes the procedure which is used to compute the reliability index β:

(1) Define the limit state equation.

(2) Make an initial guess at the reliability index β.

(3) Set the initial design point values $x_i^* = \bar{x}_i$, for all i.

(4) Compute the mean and standard deviation of the equivalent normal distribution for those variables that are non-normal according to eqn (35).

(5) Transform variables x_i to reduced variables u_i and $g(x_1, \ldots, x_n)$ to $g_1(u_1, \ldots, u_n)$.

(6) Compute partial derivatives $\partial g_1/\partial u_i$ evaluated at the point u_i^*.

(7) Compute the direction cosines α_i from eqn (29).

(8) Compute new values of u_i^* from eqn (30).

(9) Repeat steps 4 through 7 until the estimates of α_i stabilize.

(10) Compute the value of β necessary for $g_1(u_1^*, \ldots, u_n^*) = 0$.

(11) Repeat steps 4 through 10 until the values of β on successive iterations differ by some small tolerance (say 0.01).

(12) Compute failure probability using eqn (34).

In view of eqn (11), the limit state equation for fatigue reliability analysis is expressed as

$$g(N_1, \ldots, N_k) = 1 - D = 0 \tag{36}$$

Herein only fatigue lives (N_i) for cyclic stresses of constant stress amplitudes are treated as random variables. It is noted that the above limit state equation when derived from the forementioned cumulative damage models can be expressed explicitly in terms of the random variables N_1, N_2, ..., N_k and thus the adopted reliability evaluation method can be applied directly to the fatigue reliability analysis without going through any transformation. The shapes of the limit state equations constructed on the basis of the above cumulative damage models with two stress levels are similar to the one shown in Fig. 1. It

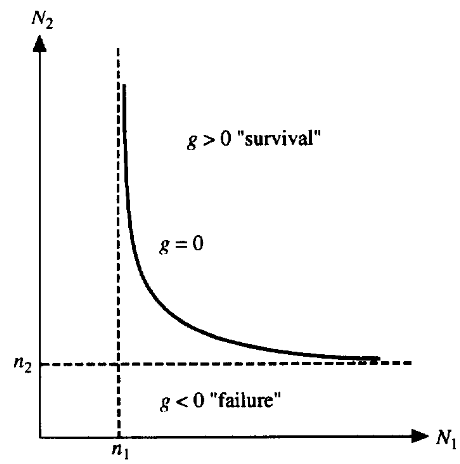

Fig. 1. Failure curve for two-stress level fatigue.

is worth noting that both Palmgren–Miner's and modified Palmgren–Miner's models yield the same limit state equation and thus same reliability as expected. Using the equivalent damage concept, the limit state equations for the four cumulative damage models under two or three stress levels can be derived from eqns (15)–(22) and written as:

(a) *Palmgren–Miner's model.* Two-stress level

$$1-\left[\frac{n_1}{N_1}+\frac{n_2}{N_2}\right]=0; \tag{37}$$

three-stress level

$$1-\left[\frac{n_1}{N_1}+\frac{n_2}{N_2}+\frac{n_3}{N_3}\right]=0. \tag{38}$$

(b) *Modified Palmgren–Miner's model.* Two-stress level

$$1-\left(\frac{n_1}{N_1}+\frac{n_2}{N_2}\right)^c=0; \tag{39}$$

three-stress level

$$1-\left(\frac{n_1}{N_1}+\frac{n_2}{N_2}+\frac{n_3}{N_3}\right)^c=0. \tag{40}$$

(c) *H-H-I.* Two-stress level

$$1-\left[\frac{\mathrm{B}-N_1^c}{\mathrm{B}-n_1^c}+\frac{\mathrm{B}-N_2^c}{\mathrm{B}-(n_{12}+n_2)^c}-\frac{\mathrm{B}-N_2^c}{\mathrm{B}-n_{12}^c}\right]=0; \tag{41}$$

three-stress level

$$1-\left[\frac{\mathrm{B}-N_1^c}{\mathrm{B}-n_1^c}+\frac{\mathrm{B}-N_2^c}{\mathrm{B}-(n_{12}+n_2)^c}-\frac{\mathrm{B}-N_2^c}{\mathrm{B}-n_{12}^c}+\frac{\mathrm{B}-N_3^c}{\mathrm{B}-(n_{23}+n_3)^c}-\frac{\mathrm{B}-N_3^c}{\mathrm{B}-n_{23}^c}\right]=0. \tag{42}$$

(d) *H-H-II.* Two-stress level

$$1-\left\{\left(\frac{n_1}{N_1}\right)^c\left[\frac{\mathrm{B}-N_1^c}{\mathrm{B}-n_1^c}\right]+\left(\frac{n_{12}+n_2}{N_2}\right)^c\left[\frac{\mathrm{B}-N_2^c}{\mathrm{B}-(n_{12}+n_2)^c}\right]-\left(\frac{n_{12}}{N_2}\right)^c\left[\frac{\mathrm{B}-N_2^c}{\mathrm{B}-n_{12}^c}\right]\right\}=0; \tag{43}$$

three-stress level

$$1-\left\{\left(\frac{n_1}{N_1}\right)^c\left[\frac{\mathrm{B}-N_1^c}{\mathrm{B}-n_1^c}\right]+\left(\frac{n_{12}+n_2}{N_1}\right)^c\left[\frac{\mathrm{B}-N_2^c}{\mathrm{B}-(n_{12}+n_2)^c}\right]-\left(\frac{n_{12}}{N_2}\right)^c\left[\frac{\mathrm{B}-N_2^c}{\mathrm{B}-n_{12}^c}\right]\right.$$
$$\left.+\left(\frac{n_{23}+n_3}{N_3}\right)^c\left[\frac{\mathrm{B}-N_3^c}{\mathrm{B}-(n_{23}+n_3)^c}\right]-\left(\frac{n_{23}}{N_3}\right)^c\left[\frac{\mathrm{B}-N_3^c}{\mathrm{B}-n_{23}^c}\right]\right\}=0. \tag{44}$$

4. EXPERIMENTAL INVESTIGATION

For specimen preparation, graphite/epoxy [(Q-1115) supplied by Toho Co., Japan] prepreg tape was used to make eight-layer $[45°/-45°_2/45°]_s$ composite laminates. The

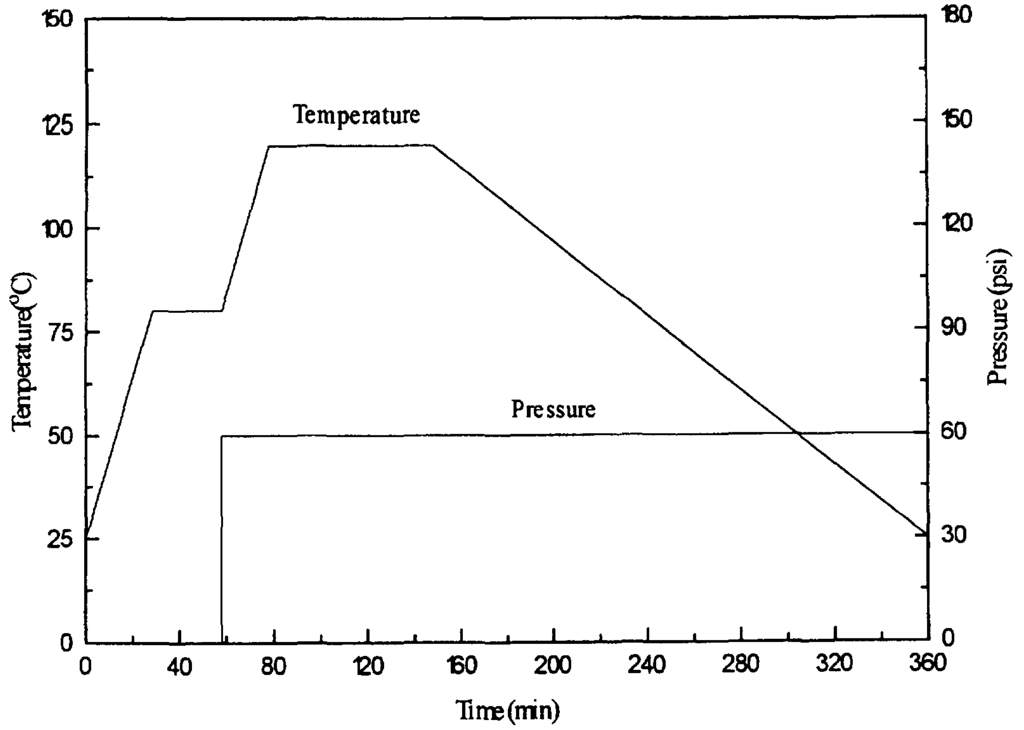

Fig. 2. Cure cycle for graphite/epoxy laminates (1 psi = 682.66 Pa).

laminates were cured by a hot press machine via the curing process shown in Fig. 2. Each cured laminate was saw cut into 10 specimens with dimensions shown in Fig. 3 and three out of the 10 laminated specimens were subjected to tensile test for determining the mean ultimate strength $\bar{S}_u$ of the laminates. It is noted that for the same laminate the variation of ultimate strength was small and in general the coefficient of variation was less than 0.2%. A typical load–stroke relation of the specimens is shown in Fig. 4.

Fatigue life tests of the $[45°/-45°_2/45°]_s$ specimens were performed under the isothermal condition using a 10-ton Instron testing machine. A set of (14–16) specimens was tested to complete failure at the frequency of 2 Hz for each of the following stress levels: $r = 0.95$, 0.9 and 0.85, with mean stress $S_a = 0.5\bar{S}^i_u$ where $\bar{S}^i_u$ is the mean of ultimate strength of the

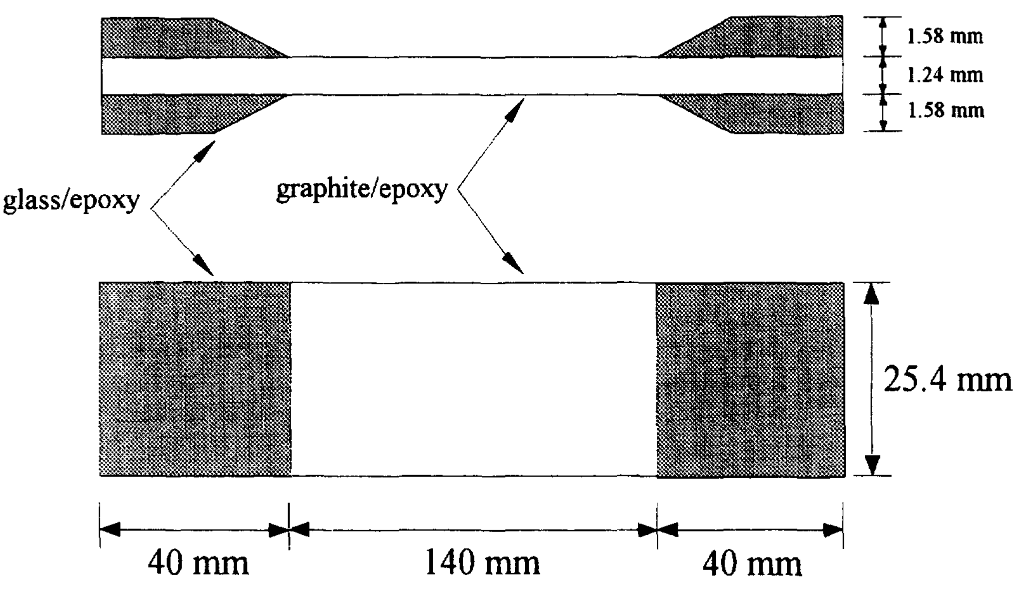

Fig. 3. Test specimen configuration.

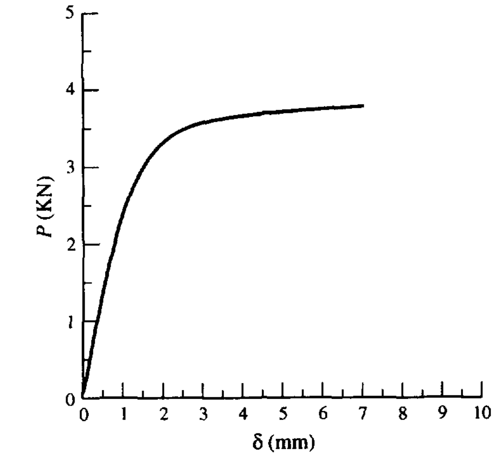

Fig. 4. Load-stroke relation of $[45°/-45°_2/45°]_s$ graphite/epoxy specimen.

ith laminate. The fatigue life data for each of the stress levels were fitted by either lognormal (see Figs 5–7) or Weibull distributions (see Figs 8–10) on probability papers via the least square method. It is noted that in general both lognormal and Weibull distributions can yeild good fit of the test data. The probability density functions of lognormal and Weibull are expressed, respectively, as:

Lognormal distribution

$$f_N(n) = \frac{1}{n\sqrt{2\pi}\xi}\exp\left[-\left(\frac{\ln(n)-\lambda}{\xi}\right)^2\right] \tag{45}$$

where $f_N(n) \geqslant 0$, $n > 0$; λ is mean of natural logarithm of cycles to failure and ξ is standard deviation of natural logarithm of cycles to failure.

Weibull distribution

$$f_N(n) = \frac{\alpha}{\theta}\left(\frac{n-\gamma}{\theta}\right)^{\alpha-1}\exp\left[-\left(\frac{n-\gamma}{\theta}\right)^{\alpha}\right] \tag{46}$$

where $f_N(n) \geqslant 0$, $n \geqslant 0$; α is shape parameter, θ is scale parameter and γ is location parameter or minimum fatigue life. Herein, for simplicity γ is assumed to be zero. The statistics, mean and coefficient of variation and parameters of the fatigue life distributions for the three constant stress levels are listed in Table 1. It is noted that the fatigue life of the composite material has very large variation (coefficient of variation is greater than 40%). A similar phenomenon has also been observed by Yang *et al.* (1992) in which the coefficient of variation was great than 102%. The mean lives of the laminates at various stress levels are used to determine the constants, $B = 91.6$ and $c = 0.306$, in the Hwang–Han's models of eqns (7) and (9) via the least square method. Fatigue life tests of the $[45°/-45°_2/45°]_s$ specimens subjected to spectrum stresses of two or three stress levels were also conducted and the numbers of cycles at different stress levels were recorded for the following cases: (i) record the number of cycles n_2 for stress level $r_2 = 0.95$ after the specimens have been tested for $n_1 = 200$ at the stress level $r_1 = 0.9$; (ii) record the number of cycles n_2 for stress level $r_2 = 0.9$ after the specimens have been tested for $n_1 = 80$ at the stress level $r_1 = 0.95$; (iii) record the number of cycles n_3 for $r_3 = 0.85$ after the specimens have been tested for $n_1 = 80$ and $n_2 = 150$ at $r_1 = 0.95$ and $r_2 = 0.9$, respectively; (iv) record the number of cycles n_3 for $r_3 = 0.95$ after the specimens have been tested for $n_1 = 1500$ and $n_2 = 150$ at $r_1 = 0.85$ and $r_2 = 0.9$, respectively. The fatigue life data were fitted by either lognormal (see Figs 11–14) or Weibull distributions (see Figs 15–18). Again it is noted that in general both lognormal and Weibull distributions can yield good fit of the test data except for cases

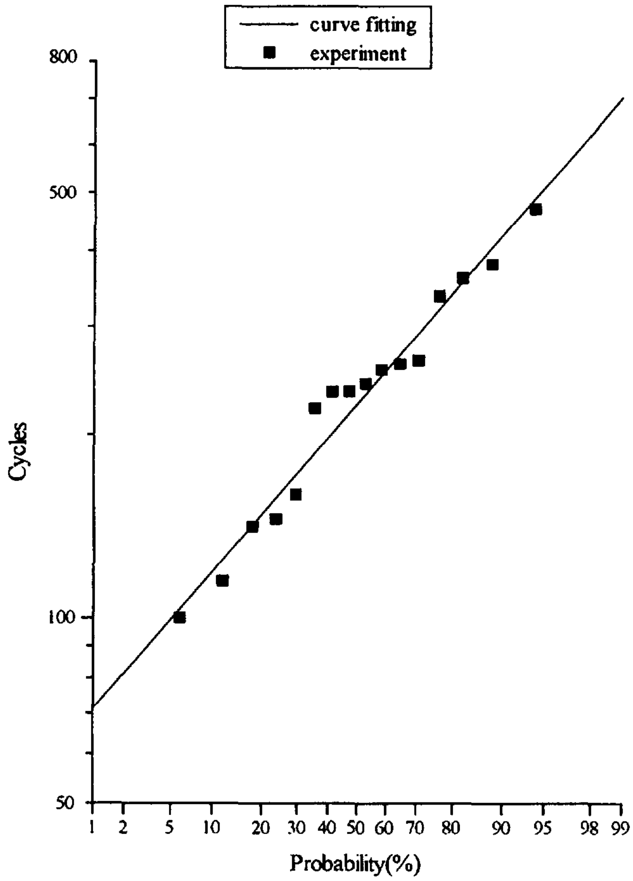

Fig. 5. Fatigue life data for $r = 0.95$ fitted by lognormal distribution.

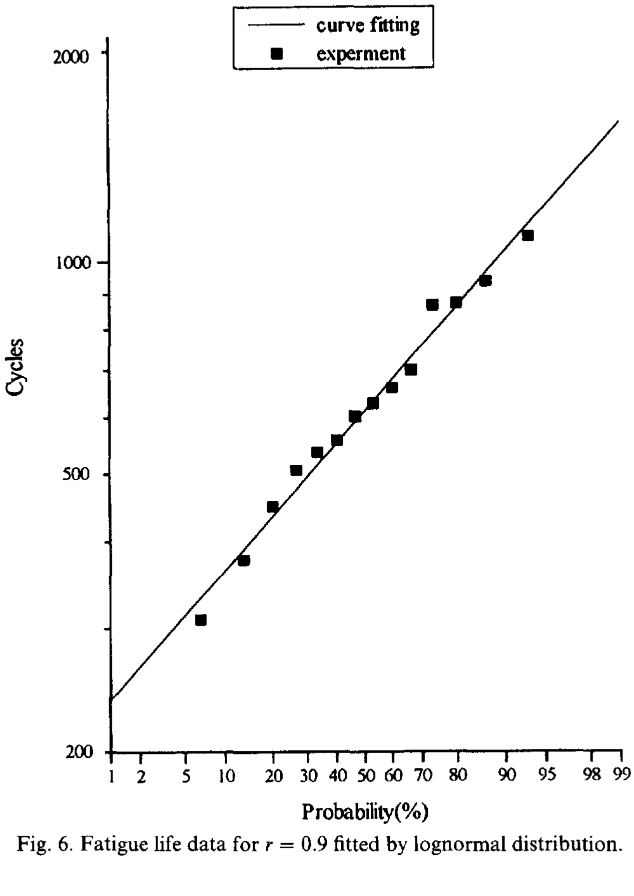

Fig. 6. Fatigue life data for $r = 0.9$ fitted by lognormal distribution.

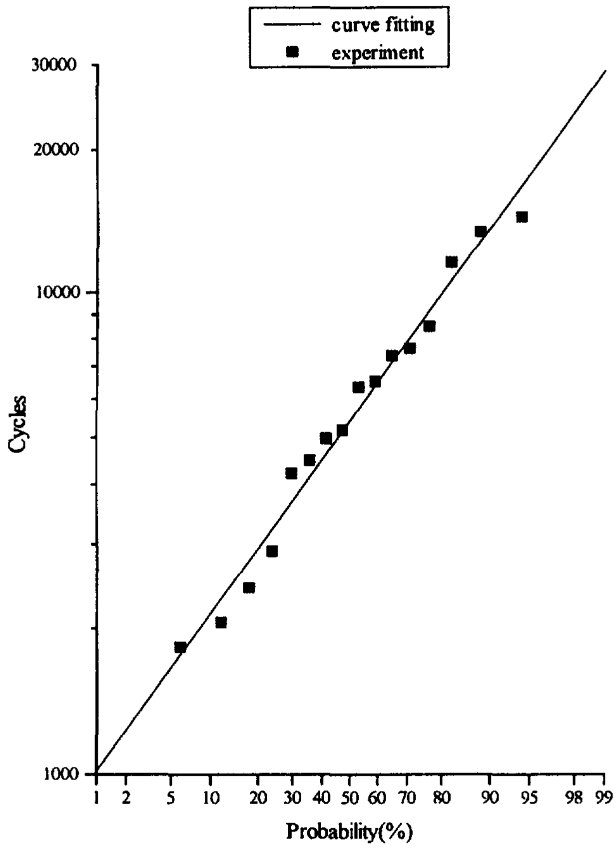

Fig. 7. Fatigue life data for $r = 0.85$ fitted by lognormal distribution.

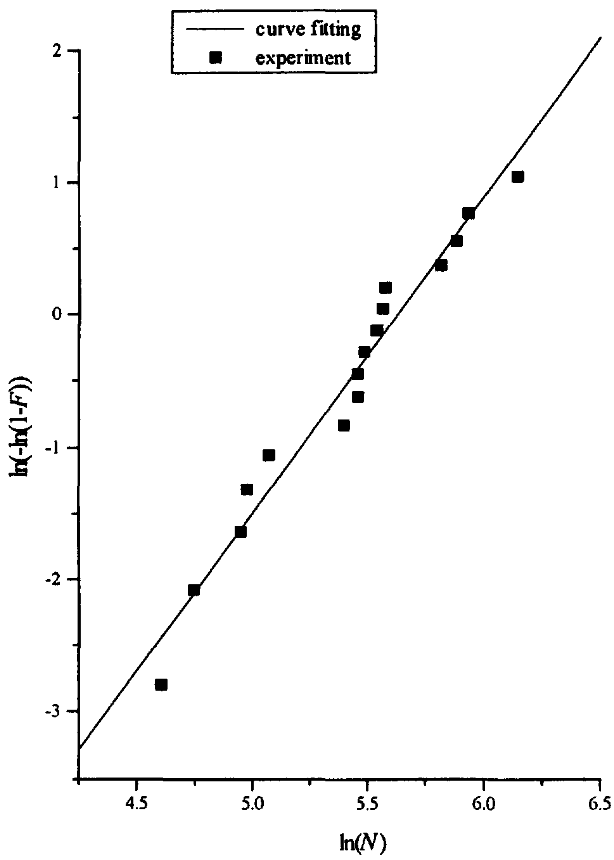

Fig. 8. Fatigue life data for $r = 0.95$ fitted by Weibull distribution.

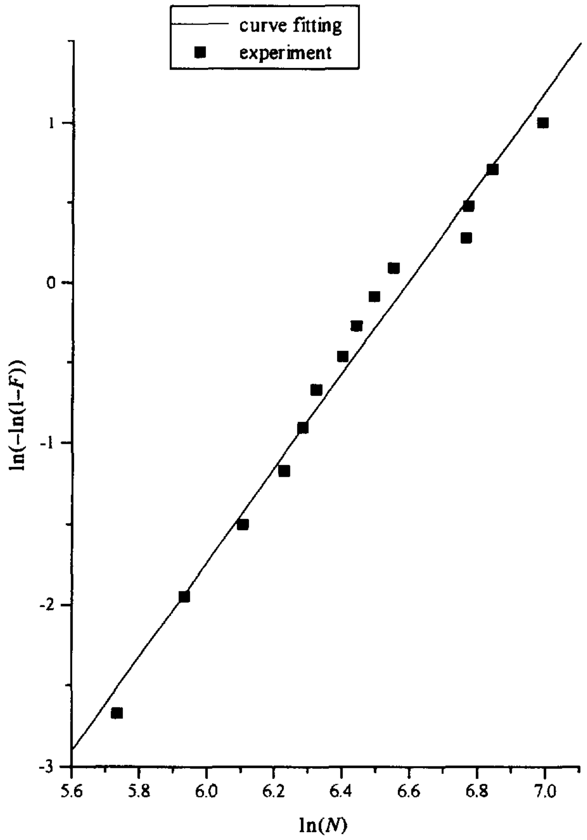

Fig. 9. Fatigue life data for $r = 0.9$ fitted by Weibull distribution.

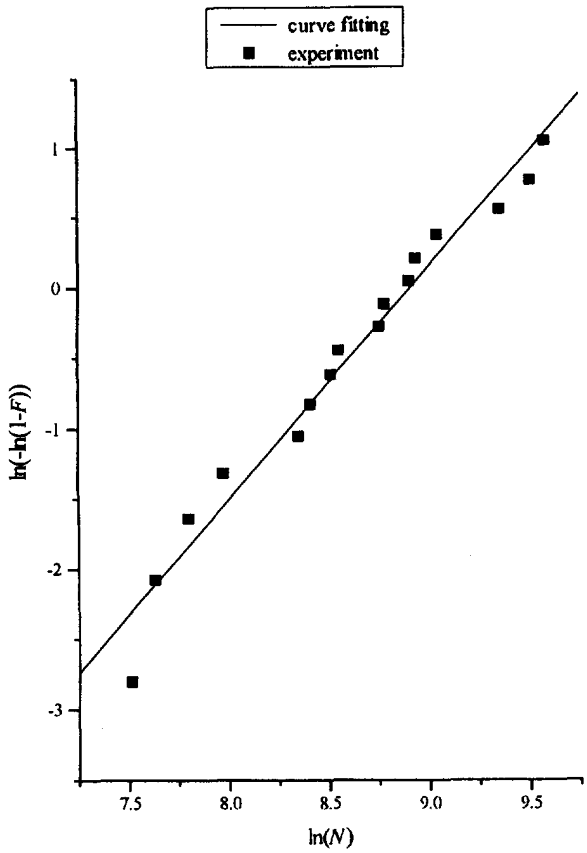

Fig. 10. Fatigue life data for $r = 0.85$ fitted by Weibull distribution.

Table 1. Statistics of fatigue life at various stress levels fitted by different probability distributions

	Lognormal				Weibull	
r	λ	ξ	$\bar{N}$	c.o.v. = $\sigma/\bar{N}$	α	Θ
0.85	8.59917	0.64685	6690.06	0.7208	1.64566	7420.208
0.90	6.41836	0.39224	662.165	0.4077	2.92501	729.77
0.95	5.41079	0.46561	249.43	0.4920	2.38782	277.653

Table 2. Statistics of fatigue life distributions under various spectrum stresses

	Lognormal		Weibull	
Case	Median	c.o.v	α	Θ
(i) $r_1 = 0.90$, $r_2 = 0.95$	170.33	0.6649	1.64252	232.695
(ii) $r_1 = 0.95$, $r_2 = 0.90$	484.03	0.85925	1.38578	701.873
(iii) $r_1 = 0.85$, $r_2 = 0.90$, $r_3 = 0.95$	91.03	0.99809	1.11716	143.009
(iv) $r_1 = 0.95$, $r_2 = 0.90$, $r_3 = 0.85$	2293.72	0.83382	1.43808	3275.923

(i) and (iv) in which distortions of the test data are obvious. The statistics and parameters of the probability distributions are given in Table 2. It is again noted that the variations of fatigue lives for the above cases are very large (coefficient of variations are greater than 66%). The residual fatigue lives of the laminates under various spectrum stresses with reliability, say, $P_S = 0.9$ can be determined directly from Figs 11–18 and the results are listed in Table 3.

5. RESULTS AND DISCUSSIONS

The feasibility of the aforementioned technique, namely, modified β-method and the applicability of the adopted cumulative damage models for fatigue reliability analysis of composite materials under spectrum stress, is studied using the test data given in the previous section. In the reliability analysis, the fatigue lives $N_1, N_2, \ldots, N_k$ at various stress levels of constant amplitudes are treated as independent random variables. Based on the fatigue life data of the laminates under cyclic stress and the probability distributions given in Table 1, the theoretical reliabilities of the laminates under different spectrum stresses are evaluated using the above various limit state equations and the residual lives listed in Table 3 via the modified β-method. The theorectical predictions are listed in Tables 4–5 in comparison with the target reliability (0.9) obtained from experimental data. It has been shown that irrespective of stress sequence the theoretical approach can yield reasonably good results for the laminates provided that fatigue damages are modeled by Palmgren–Miner's rule or H-H-II and fatigue lives simulated by lognormal distribution. Furthermore, among the adopted cumulative damage models the Palmgren–Miner's rule can yield results with consistent accuracy no matter whether the fatigue lives are simulated by either log-normal or Weibull distributions. On the contrary, H-H-I cannot yield consistent accuracy in fatigue reliability prediction and thus it may not be applicable for fatigue reliability analysis of composite laminates. It is also worth noting that stress sequence effect is not obvious because the accuracy of the results predicted by the present approach does not have any direct correlation to the sequence of applied stress levels.

To further demonstrate the advantages of using the present approach for fatigue reliability analysis, it is worth studying the difference between the present and the so called "conventional" approach in fatigue life prediction for achieving the same target reliability.

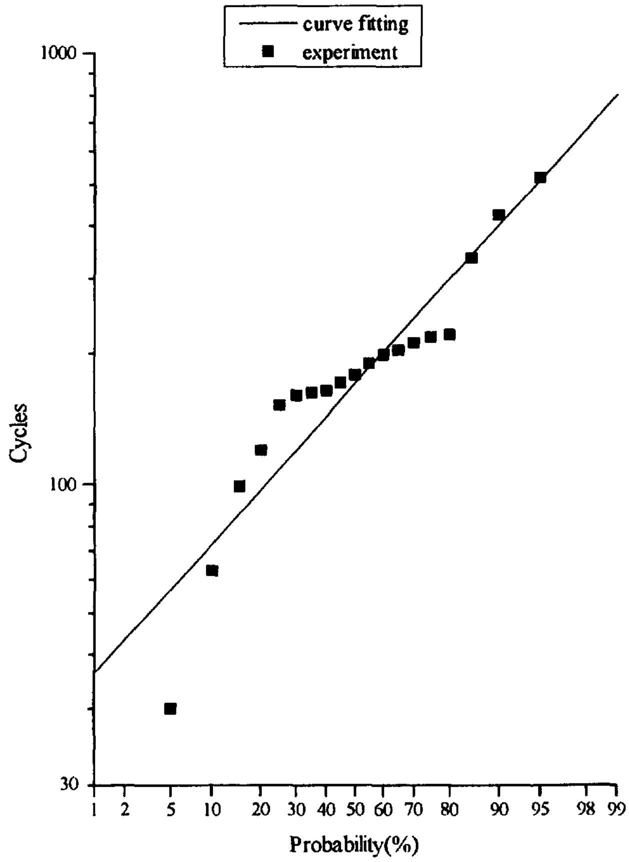

Fig. 11. Residual fatigue life n_2 fitted by lognormal distribution for $r_2 = 0.9$, $n_1 = 200$ and $r_2 = 0.95$.

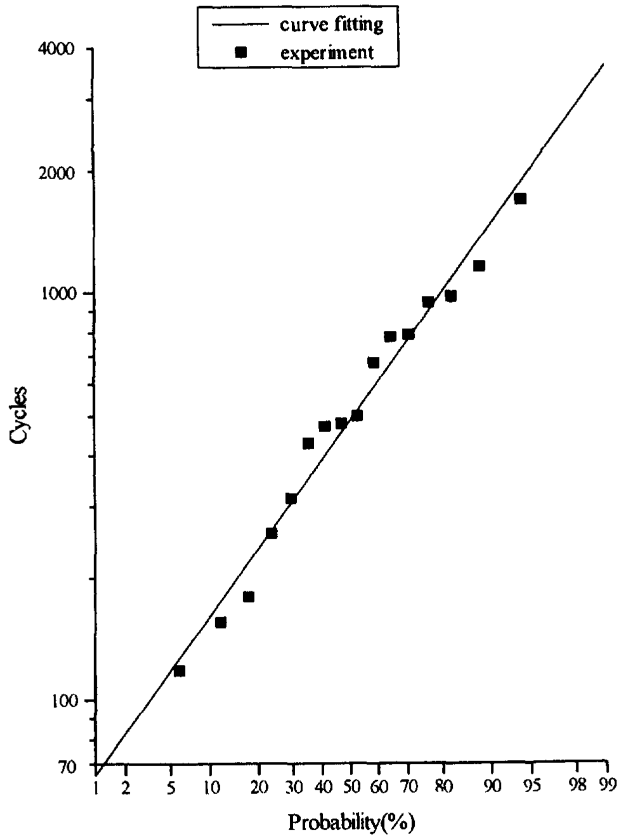

Fig. 12. Residual fatigue life n_2 fitted by lognormal distribution for $r_2 = 0.95$, $n_1 = 80$ and $r_2 = 0.9$.

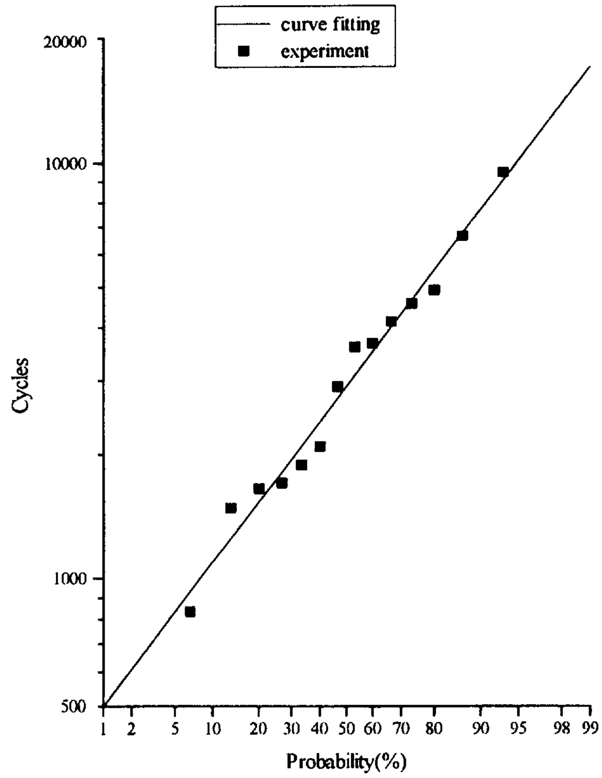

Fig. 13. Residual fatigue life n_3 fitted by lognormal distribution for $r_1 = 0.95$, $n_1 = 80$, $r_2 = 0.9$, $n_2 = 150$, $r_3 = 0.85$.

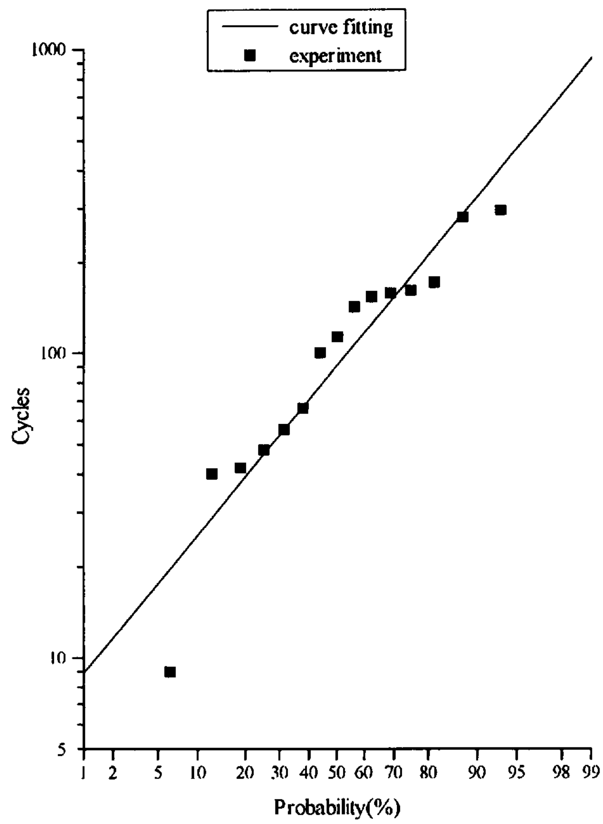

Fig. 14. Residual fatigue life n_3 fitted by lognormal distribution for $r_1 = 0.85$, $n_1 = 1500$, $r_2 = 0.9$, $n_2 = 150$, $r_3 = 0.95$.

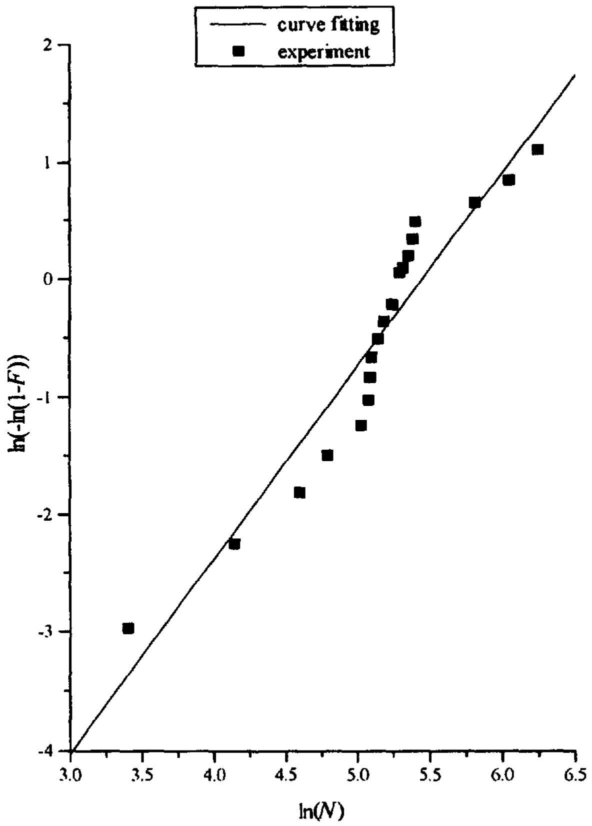

Fig. 15. Residual fatigue life n_2 fitted by Weibull distribution for $r_1 = 0.9$, $n_1 = 200$ and $r_2 = 0.95$.

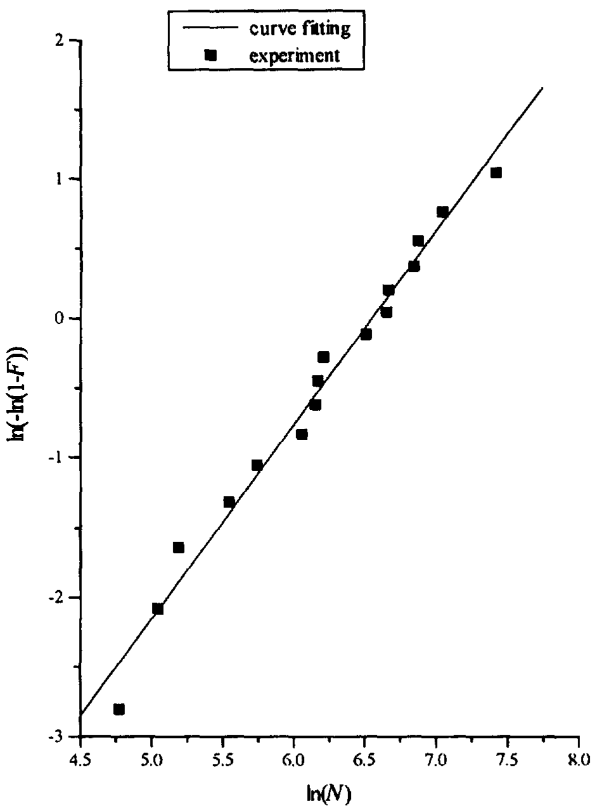

Fig. 16. Residual fatigue life n_2 fitted by Weibull distribution for $r_1 = 0.95$, $n_1 = 80$ and $r_2 = 0.9$.

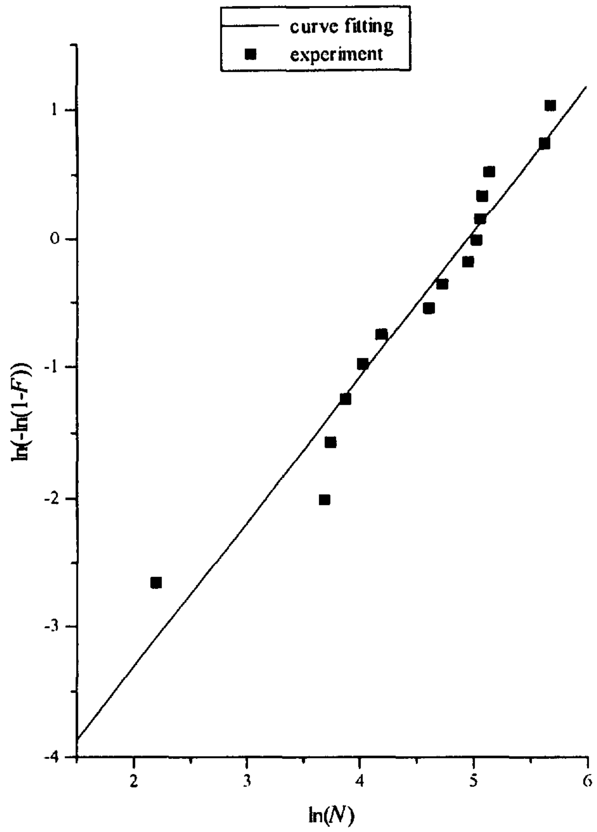

Fig. 17. Residual fatigue life n_3 fitted by Weibull distribution for $r_1 = 0.85$, $n_1 = 1500$, $r_2 = 0.9$, $n_2 = 150$, $r_3 = 0.95$.

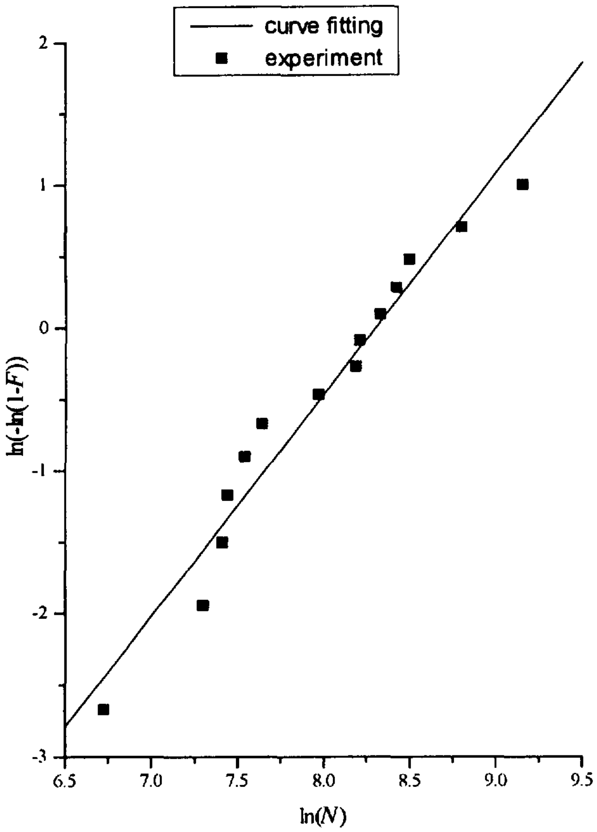

Fig. 18. Residual fatigue life n_3 fitted by Weibull distribution for $r_1 = 0.95$, $n_1 = 80$, $r_2 = 0.9$, $n_2 = 150$, $r_3 = 0.85$.

Table 3. Fatigue lives with reliability 0.9 under various spectrum stresses

		n_2		n_3	
Spectrum stress	n_1	Lognormal	Weibull	Lognormal	Weibull
$r_1 = 0.90, r_2 = 0.95$	200	72.65	59.121	—	—
$r_1 = 0.95, r_2 = 0.90$	80	160	138.36	—	—
$r_1 = 0.85, r_2 = 0.90, r_3 = 0.95$	1500	150	150	25.33	19.078
$r_1 = 0.95, r_2 = 0.90, r_3 = 0.85$	80	250	150	787.88	685.06

In the conventional approach (Collins, 1981), when the target reliability α is prescribed, the fatigue life of a laminate subjected to spectrum stress can be determined from any cumulative damage model, say, Palmgren–Miner's rule, via the following procedure. Let $N_{k,\alpha}$ be the fatigue life of reliability α subject to the kth stress level. The allowable applied stress cycles at different stress levels are determined by solving the following equation:

$$\sum_{i=1}^{k} \frac{n_i}{N_{i,\alpha}} = 1. \tag{47}$$

Assuming that fatigue life is lognormally distributed, the fatigue lives of the laminates with reliability $\alpha = 0.9$ at different stress levels can then be determined from Figs 5–7 as

$$\begin{aligned} r_1 &= 0.85; \quad N_{1,0.9} = 2155; \\ r_2 &= 0.9; \quad N_{2,0.9} = 364; \\ r_3 &= 0.95; \quad N_{3,0.9} = 119. \end{aligned} \tag{48}$$

Using the above fatigue life data, the residual lives of the laminates subjected to the four different stress sequences as described in the previous section are determined from eqn (47). The residual lives predicted by the conventional approach are listed in Table 6 in comparison with those obtained from experiments as well as those predicted by the present approach. It is noted that the conventional approach may yield erroneous results to such extent that residual life becomes negative while both the experimental and the present approaches yield non-negative residual lives. Furthermore, the present approach can yield reasonably good results when compared with the experimental ones as shown in Table 6.

6. CONCLUSIONS

A systematic approach was developed to study the reliability of graphite/epoxy composite laminates subjected to spectrum stresses. Experiments were performed to investigate the fatigue life distributions of the laminates under various spectrum stresses. The suitability of several cumulative damage models in predicting fatigue damage and reliability was studied. In this study, it has been found that both lognormal and Weibull distributions are appropriate for modeling fatigue life distribution of composite materials subjected to either cyclic stress of constant amplitude or spectrum stress. The modified β-method constructed on the basis of structural reliability theory can yield reasonably good results on fatigue reliability of composite materials provided that an appropriate cumulative damage model is adopted in the analysis. Stress sequence effect was not obvious as was observed in this study and thus may not be an important factor in fatigue reliability analysis of composite laminates. The inaccuracy of the conventional approach in fatigue reliability prediction was demonstrated and the superiority of the present approach manifested.

Table 4. Theoretical predictions of fatigue reliability based on various cumulative damage models (lognormal distribution)

Spectrum stress	Reliability (experiment)	Reliability (theoretical)				Error (%)			
	I.	II. Palmgren-miner	III. Modified-miner	IV. H-H-I	V. H-H-II	$\left\lvert\frac{I-II}{I}\right\rvert \times 100$	$\left\lvert\frac{I-III}{I}\right\rvert \times 100$	$\left\lvert\frac{I-IV}{I}\right\rvert \times 100$	$\left\lvert\frac{I-V}{I}\right\rvert \times 100$
$r_1 = 0.9$, $r_2 = 0.95$ (low–high)	0.9	0.919	0.919	0.965	0.923	2.111	2.111	7.222	2.556
$r_1 = 0.95$, $r_2 = 0.90$ (high–low)	0.9	0.939	0.939	0.863	0.937	4.333	4.333	4.111	4.111
$r_1 = 0.85$, $r_2 = 0.90$, $r_3 = 0.95$ (low–high)	0.9	0.916	0.916	0.964	0.922	1.778	1.778	7.111	2.444
$r_1 = 0.95$, $r_2 = 0.90$, $r_3 = 0.85$ (high–low)	0.9	0.863	0.863	0.492	0.985	4.111	4.111	45.33	9.411

Table 5. Theoretical predictions of fatigue reliability based on various cumulative damage models (Weibull distribution)

Spectrum stress	Reliability (experiment)	Reliability (theoretical)				Error (%)			
	I.	II. Palmgren-miner	III. Modified-miner	IV. H-H-I	V. H-H-I	$\left\lvert\frac{I-II}{I}\right\rvert \times 100$	$\left\lvert\frac{I-III}{I}\right\rvert \times 100$	$\left\lvert\frac{I-IV}{I}\right\rvert \times 100$	$\left\lvert\frac{I-V}{I}\right\rvert \times 100$
$r_1 = 0.9$, $r_2 = 0.95$ (low–high)	0.9	0.937	0.937	0.968	0.939	4.111	4.111	7.556	4.333
$r_1 = 0.95$, $r_2 = 0.90$ (high–low)	0.9	0.953	0.953	0.907	0.951	5.889	5.889	0.778	5.667
$r_1 = 0.85$, $r_2 = 0.90$, $r_3 = 0.95$ (low–high)	0.9	0.937	0.937	0.982	0.940	4.111	4.111	9.111	4.444
$r_1 = 0.95$, $r_2 = 0.90$, $r_3 = 0.85$ (high–low)	0.9	0.903	0.903	0.644	0.894	0.333	0.333	28.44	0.667

Table 6. Fatigue lives of laminates under various spectrum stresses predicted by different methods for target reliability $P_s = 0.9$

Method	Two stress levels (n_2)		Three stress levels (n_3)	
	$r_1 = 0.9, r_2 = 0.95, n_1 = 200$	$r_1 = 0.95, r_2 = 0.90, n_1 = 80$	$r_1 = 0.85, r_2 = 0.90, r_3 = 0.95, n_1 = 1500, n_2 = 150$	$r_1 = 0.95, r_2 = 0.90, r_3 = 0.85, n_1 = 80, n_2 = 150$
I. Experiment	73	160	25	788
II. Present method	74	183	28	630
III. Conventional method	53	119	−13	−181
Error (%)				
$\left\lvert\frac{\text{I–II}}{\text{I}}\right\rvert \times 100$	1.370	14.375	12.00	20.051
$\left\lvert\frac{\text{I–III}}{\text{I}}\right\rvert \times 100$	27.397	25.625	152.00	122.970

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